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TECHNICAL NOTES

A composite correlation for heat transfer from isolated two- and three-dimensional protrusions in channels

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INTRODUCTION

THE INCREASING miniaturization of electronic components and resultant rise in heat flux at the chip level has motivated research aimed at improved understanding of the fundamentals of heat transfer in discrete heating situations. Recently, heat transfer and flow structure results were reported for isolated three-dimensional protrusions in channels [1]. In this case the spanwise extent of the heated module was smaller than the channel width, $P_w < W$, as shown in Fig. 1. The objective of this Technical Note is to present complementing *two-dimensional* protrusion heat transfer results, for which $P_w = W$. It is also the goal to develop a composite correlation for all two- and three-dimensional module data. The experimental data serve to elucidate physical trends for heat transfer from finite protrusions in channels, and also to provide a data base for comparison with theoretical models under development.

Heat transfer characteristics of discretely heated protrusions in forced convection situations have been treated to some extent in the literature. Heat transfer from two-dimensional protrusions in channels was studied experimentally [2–6], and three-dimensional arrays of heated protrusions have also been investigated [7–9]. The influence of protrusion three-dimensionality in heated module arrays has also been treated [10–12]. The problem of forced convection from discretely heated components has also received attention from the analytical perspective in laminar two-dimensional [13], turbulent two-dimensional [14, 15], and laminar three-dimensional [16] geometries using finite difference techniques. In all of these studies intimate coupling between the flow structure and the heat transfer was observed, and was described by correlations expressed in terms of the geometric parameters of the problem.

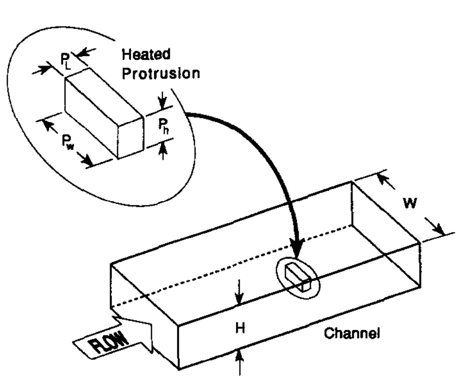

FIG. 1. Schematic of system under study and definition of geometric parameters.

EXPERIMENTAL APPARATUS AND INSTRUMENTATION

A channel and heater assembly were designed and constructed to permit heat transfer measurements over a broad range of experimental parameters. A complete description of the experimental apparatus is given elsewhere [1], and will therefore only be summarized here. The channel cross-section was divided symmetrically in half by a thin, internally-ribbed cardboard divider, forming identical top and bottom channels (see Fig. 2). Each of the two channels was equipped with an instrumented heater module for measurement, the symmetric channel minimizing heat losses through the channel floor. The channel was designed to have variable height capability. The channel height was adjusted by means of moveable channel walls. Based on the relationship for the entrance length corresponding to fully-developed flow [17], the channel was constructed to be 1520 mm in overall length, allowing approximately 200 mm between the heated protrusion and the channel exit.

As explained in the foregoing, high thermal conductivity modules, heated electrically at the same rate, were placed on opposite sides of the channel divider. A cross-sectional detail of the heater blocks is shown in the lower panel of Fig. 2. A small coil of nichrome wire, which was connected to electrical leads, was designed to fit inside a section milled in each module, and was used for ohmic heating. Small voltage sensing leads were attached to the nichrome coil at the junction with the power leads, and were used to measure the voltage drop across each coil during each experiment. The thermocouple, power supply, and voltage sensing leads were drawn out of the channel toward the downstream plenum between the ribs on the interior of the cardboard channel divider. Removeable cardboard sections with identical heater modules on top and bottom were constructed for the two dimensional protrusions ($P_w = W$) of the same streamwise length $P_L = 12$ mm. Four different gap spacings between the protrusion and channel ceiling, $(H - P_h) = 3$, 8, 12, and 18 mm, were investigated for each of three protrusion heights, $P_h = 4$, 8, and 12 mm, and the results are reported here. Thus, the channel wall spacing was varied in the range $7 \leqslant H \leqslant 30$ mm. Three-dimensional protrusions ($P_w < W$), also of streamwise length $P_L = 12$ mm, were studied pre-

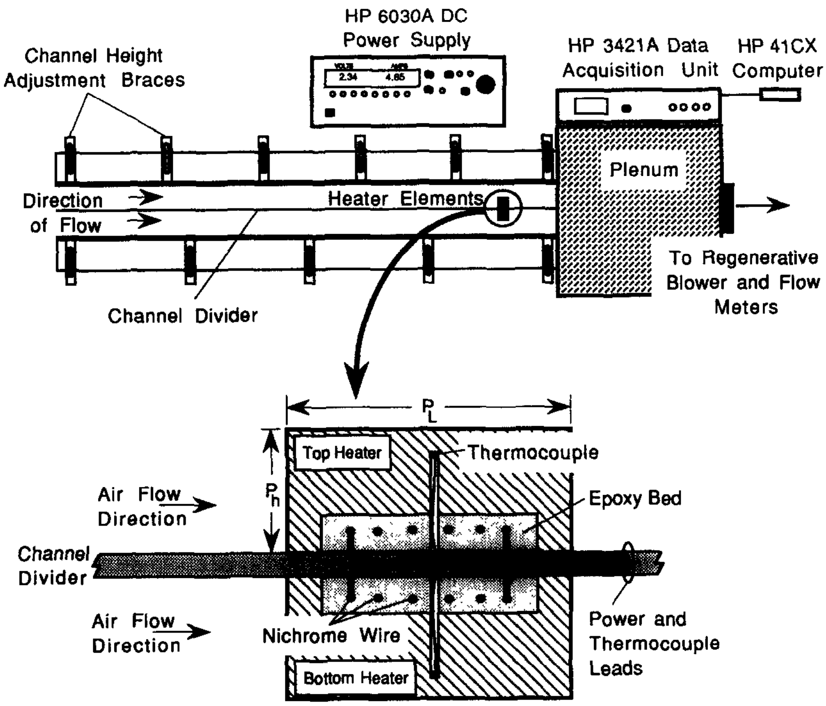

FIG. 2. Schematic of experimental apparatus and heater modules.

viously for a fixed module height of $P_h = 12$ mm and variable module width $P_w = 19$, 38, and 75 mm [1].

Five copper/constantan thermocouples were embedded in both top and bottom heater modules for determining the average temperature, $\bar{T}_s$. The variation in the temperature measurements on the module interior was never more than 6% of the difference between the average module and free-stream temperature, $\bar{T}_s - T_\infty$. Additionally, the variation in $\overline{Nu}$ between top and bottom heaters was always less than 5%, with the average difference over all experiments being less than 2%. A copper/constantan (Cu/Co) thermocouple was placed at the channel entrance to measure the inlet temperature (T_∞). The protrusions were constructed of 6061-T6 aluminum alloy ($k = 172$ W m^{-1} K^{-1}). The exposed aluminum surfaces were highly polished to reduce the heat transfer by radiation, and the emissivity was measured using infrared radiometry to be approximately 0.05. The maximum and average radiation heat loss for all experiments were 1.4% and 0.5%, respectively, of the total heat transfer.

Because of the dual channel symmetry, each heater module acted as a guard heater for the one opposite. However, an analysis of the conjugate heat transfer in the channel divider revealed that the conduction loss was non-negligible. To estimate the heat loss, the channel divider was approximated as an annular fin whose base temperature was equal to the heated module temperature. The heat transfer from the heater was then determined as the ohmic dissipation minus the conduction and radiation losses.

The room air was drawn into the channel entrance over the heated modules in the test section, into the plenum, and out through bypass flow meters to the regenerative blower, which was operated in suction mode. Three rotameters with different flow rate measurement ranges 14.2–70.8, 85.0–396.4, and 340.0–1529.1 l m^{-1} were used to meter the air flow rate. The flow meters have a calibrated accuracy of 5%. Since the upper and lower channels were constructed to be symmetric in every way, it was assumed that the global and local flow characteristics of the upper and lower channels were identical. This is supported by the fact that the maximum difference between the average Nusselt number for the upper and lower heaters was only 4.0% in all of the experiments. Indeed, the average difference between upper and lower heated module Nusselt numbers was less than 2%. This is well within the experimental uncertainty in $\overline{Nu}$ of 7.4% to be presented.

An error analysis using the method of Kline and McClintock [18] was performed to determine the uncertainty in the experimental data, and is presented in detail elsewhere [1]. In summary, the maximum composite uncertainty in average Nusselt number was found to be 7.4%, while the uncertainty in Reynolds number of 7.7% was estimated. This number includes an estimate of asymmetry in flow partitioning between the parallel channels in the experimental system (based on the average 2% difference in Nusselt number for the top and bottom heaters).

EXPERIMENTAL RESULTS

The parameters for the convective heat transfer from the protrusions were characterized in non-dimensional form by the average Nusselt number, $\overline{Nu} = \bar{h}P_L/k$, and the channel Reynolds number, $Re = UD_H/\nu$. Here, U is the average channel velocity upstream of the heated protrusion and D_H is the unobstructed channel hydraulic diameter. The average heat transfer coefficient, $\bar{h}$, was determined from the corrected heat transfer from the heater module, q, the average heater module temperature, $\bar{T}_s$, and the total convective surface area of the heater, as $\bar{h} = q/A_s(\bar{T}_s - T_\infty)$. The surface area A_s was determined as $2P_hP_w + P_LP_w + 2P_hP_L$, where the $2P_hP_L$ area term corresponding to the protrusion sides parallel to

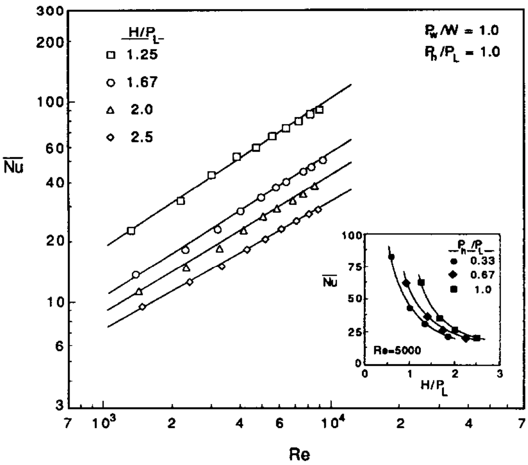

FIG. 3. Variation of average Nusselt number with Reynolds number for the $P_h/P_L = 1.0$ configuration.

the primary flow is dropped for the two-dimensional heaters. The streamwise length of the protrusion was chosen as the characteristic length in all dimensionless variables except the channel Reynolds number, which was based on the channel hydraulic diameter. Hence, the dimensionless channel height and protrusion height are normalized as H/P_L and P_h/P_L, respectively. The three-dimensional module widths, for which results were reported previously, were normalized with W as P_w/W. Note that for a fixed channel width and protrusion length, each P_w/W has a unique P_w/P_L counterpart. The experimental study included the effects of channel Reynolds numbers in the nominal range $1500 \leqslant Re \leqslant 10\,000$, protrusion heights varying between $0.33 \leqslant P_h/P_L \leqslant 1.0$, and normalized channel height in the range $0.583 \leqslant H/P_L \leqslant 2.5$. The normalized spacing between the heated module and the channel ceiling was varied between $0.25 \leqslant (H-P_h)/P_L \leqslant 1.5$. In all, over 100 experiments were conducted with the two-dimensional heaters to assess the effect of the various operating parameters on the average heat transfer.

Figure 3 illustrates the variation of the average Nusselt number with channel Reynolds number for the $P_h/P_L = 1.0$ module with various channel wall spacings. These heat transfer data are representative of the other protrusion heights studied [19]. The figure shows a least-squares correlation of the experimental data for each channel wall spacing using a relationship of the form

$$\overline{Nu} = a\,Re^b. \tag{1}$$

Table 1 lists the coefficients of the correlation, a and b, for the experimental configurations studied. The maximum average deviation between correlation and experimental data

Table 1. Coefficients for the average Nusselt number correlation, equation (1)

	$P_h/P_L = 0.33$		$P_h/P_L = 0.67$		$P_h/P_L = 1.0$	
$(H-P_h)/P_L$	a	b	a	b	a	b
0.25	0.034	0.91	0.149	0.71	0.130	0.72
0.67	0.034	0.83	0.059	0.74	0.057	0.74
1.0	0.045	0.76	0.050	0.73	0.053	0.72
1.5	0.077	0.67	0.058	0.69	0.076	0.65

in any one data set is 5%. The correlation is presented to more clearly illustrate the trends in the data and can be used to support the qualitative observations made relative to the dependence of the heat transfer on the problem parameters.

Figure 3 shows that the average heat transfer coefficient increases with decreasing channel wall spacing for the $P_h/P_L = 1$ heater modules. The increase in $\overline{Nu}$ with decreasing H/P_L is shown more clearly for the three heater module heights studied in the inset of Fig. 3 for a nominal Reynolds number $Re = 5000$. The observations are to be expected, since the flow is required to accelerate more over the top of the protrusion. $\overline{Nu}$ was observed to depend functionally on $(H/P_L)^c$, where the exponent c in the dependence was found to be approximately -0.65. Note from the data of Table 1 that the dependence of average Nusselt number depends roughly on $Re^{0.7}$, which is similar to the two-dimensional heated protrusion array data reported previously [2, 6].

A composite correlation of the experimental heat transfer data for all channel wall spacings, protrusion widths, and channel Reynolds numbers was formulated from the two-dimensional module data of this study, and the three-dimensional heated protrusion data presented previously [1]. The data of Fig. 3 and Table 1 illustrate that the flow constriction presented by the protruding heater strongly influences the average heat transfer due to the increased flow velocity. Hence, a parameter A^* is defined which represents the fraction of the channel cross-section open to flow:

$$A^* = 1-(P_w/W)(P_h/H). \tag{2}$$

For a fixed channel width, W, the parameter A^* is seen to vary with protrusion width and height, and channel wall spacing. Values of A^* ranged from 0.20 to 0.82 for the two-dimensional heaters. For the three-dimensional heater data reported previously [1], the flow constriction parameter fell in the range $0.61 \leqslant A^* \leqslant 0.95$. The normalized channel height is also used in the correlation. All two-dimensional module data reported here, and three-dimensional data from the previous study, were correlated with the equation

$$\overline{Nu} = 0.150 Re^{0.632}\,(A^*)^{-0.455}(H/P_L)^{-0.727}. \tag{3}$$

Equation (3) is valid for air flow with Reynolds numbers in the laminar-transition Reynolds number range $1500 \leqslant Re \leqslant 10\,000$, normalized protrusion heights in the range $0.33 \leqslant P_h/P_L \leqslant 1.0$, protrusion widths varying between $0.12 \leqslant P_w/W \leqslant 1.0$, and channel wall spacings in the range $0.583 \leqslant H/P_L \leqslant 2.5$. The empirical correlation predicts the experimental data with an average error of 6.6% and a maximum error of 27%. The correlation is illustrated with the experimental data for all two- and three-dimensional heaters in Fig. 4. In reality a correlation which included P_w/P_L and P_h/P_L as parameters yields slightly better accuracy (average error 5%), but the Nusselt number dependence on these two parameters was very weak, and does not justify the added complexity of the correlation. The strong inverse dependence on the flow restriction parameter A^* is evident in equation (3). Additionally, the average heat transfer coefficient is observed to vary inversely with dimensionless channel wall spacing, H/P_L, as was seen in Fig. 3. Although some grouping of the data is evident, the low error of the correlating equation in representing over 200 experiments attests to its accuracy.

Equation (3) reveals that $\overline{Nu}$ is proportional to Re^b, and inversely proportional to $(H/P_L)^c$ with nearly the same magnitude of exponents b and c. This suggests that the correlation could be further simplified if the average Nusselt number was expressed as a function of Reynolds number based on the streamwise length of the protrusion, $Re_{P_L} = UP_L/\nu$. The simplified correlation then takes the form

$$\overline{Nu} = 0.133(Re_{P_L})^{0.684}(A^*)^{-0.504}. \tag{4}$$

Equation (4) suffers only minor loss of accuracy over equation (3), with average and maximum error relative to all 2-D

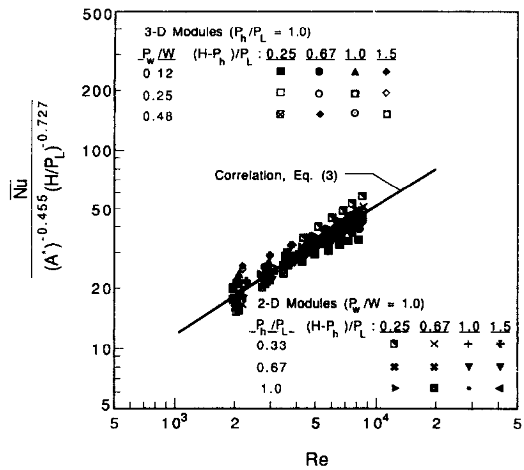

FIG. 4. Composite correlation of two-dimensional average Nusselt number data from this study and the three-dimensional data of Roeller *et al.* [1].

and 3-D experimental data of 7.2% and 25.4%, respectively. Indeed, previous investigators have found the heat transfer from arrays of two-dimensional heat sources to correlate well with a Reynolds number based on the protrusion streamwise length [2, 3]. It is also noteworthy that a previous study of heated protrusion arrays indicated only a weak dependence on channel wall spacing when the Reynolds number based on two-dimensional protrusion length was used in the correlation, with less than 15% change in $\overline{Nu}$ over the range $0.5 \leqslant H/P_L \leqslant 1.5$ [2]. Such is the finding here for isolated two- and three-dimensional protrusions.

CONCLUSIONS

The heat transfer characteristics of isolated two-dimensional protrusions in channel flow have been investigated experimentally. The parameters varied experimentally included channel Reynolds number, protrusion height, and channel wall spacing. Taller protrusions present greater channel obstruction with increased acceleration of the flow, and associated higher convective heat transfer. A composite correlation was developed for the two-dimensional data reported here and the three-dimensional protrusion data reported previously. The function describes well the average Nusselt number dependence on channel Reynolds number, normalized channel wall spacing, and a parameter quantifying the fraction of channel cross-section open to flow.

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